NEAR RINGS AND THEIR ISOTOPES

BY

BURTON VICTOR DEAN

B.S., Northwestern University, 1947
M.A., Columbia University, 1948

AN ABSTRACT OF A THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN MATHEMATICS
IN THE GRADUATE COLLEGE OF THE
UNIVERSITY OF ILLINOIS, 1952

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NEAR RINGS AND THEIR ISOTOPES

The object of this paper is to study possible generalizations of rings and ring isomorphisms. A near ring N is a system having two laws of operation, called addition and multiplication where

(1) Addition is single valued and has a zero element

(2) Multiplication is single valued and associative

(3) $ao = o = oa$ for all a in N

(4) Multiplication is left distributive with respect to addition.

It is to be noted that every ring is a near ring and all examples presented are concerned with rings. Also every system satisfying (1) - (4) is anti-isomorphic to one satisfying (1) - (3) and

(4°) Multiplication is right distributive with respect to addition.

An important example of a near ring can be constructed from an additive system S, having a zero element. The totality of all single valued functions, M(S), of S into itself, which preserve the zero element, is a near ring with multiplicative identity under the usual rules of function addition and multiplication.

(A,B,C) is an isotopy of the near ring N onto the near ring M if A,B, and C are three additive, exhaustive, 1-1 mappings of N onto M such that

$$(ab)^C = a^A b^B \text{ for all a,b in N.}$$

M is said to be an isotope of N and every isomorphism is clearly an isotopy.

The primary purposes of this paper are to find conditions for isotopic near rings to be isomorphic and to find invariant

properties of isotopic near rings. The relationship between isotopy and isomorphy is quite unique. There exist isotopic subrings of a commutative field which are not isomorphic, and even commutative rings isotopic to non-commutative rings. However, for example, if one near ring of two isotopic near rings has an identity element, then both near rings are isomorphic, and even this result can be generalized. Also, isotopes of integral domains are again integral domains and their associated quotient rings are isomorphic; again these results can be greatly generalized in the case of near rings.

In many cases it is possible to give a complete description of all isotopies of a given near ring. We consider here two results. Let (A,B,C) be an isotopy of the near ring N onto the near ring M

(a) If N has an identity element, then

$$a^A = 1^A a^D \text{ and } a^B = a^D 1^B \text{ for all } a \text{ in } N$$

where $AC^{-1}B = D = BC^{-1}A$ is an isomorphism of N onto M.

(b) If N and M are subrings of commutative fields F and K, respectively, then

$$a^A = ea^\sigma \text{ and } a^B = fa^\sigma \text{ for all } a \text{ in } N$$

where e and f are fixed elements in K and σ is an isomorphism of F onto K.

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VITA

Burton Victor Dean was born on June 3, 1924 in Chicago, Illinois. After graduating from Senn High School in 1939, he entered Wright Junior College, and received the Associate Degree from the latter institution in February, 1942. Upon completion of the Enlisted Reserve Corps program at Lewis Institute in 1943, he enlisted in the Army Signal Corps and served in the American and European Theaters. He was honorably discharged from the Army in 1946 and entered Northwestern University, where he received the Degree of Bachelor of Science in 1947. During the year 1946 to 1947 he taught at the Industrial Training Institute and American Television, Inc., both of Chicago. From 1947 through 1950 he attended the Graduate School of Columbia University and he received the Degree of Master of Arts in Mathematics in 1948 from this institution. He held Teaching Assistantships at Columbia University during 1947-1949 and at Hunter College of the City of New York during 1949-1950. In 1950 he entered the Graduate College of the University of Illinois. He held a University Fellowship from February, 1951, to June, 1952, and a Teaching Assistantship during the Summer Session, 1951. He is a member of the American Mathematical Society, Pi Mu Epsilon and Sigma Xi.

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